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A MATHEMATICAL FORMULATION FOR THE ANALYSIS OF 3-ROLL METAL FORMING PROCESSES

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A MATHEMATICAL FORMULATION FOR THE ANALYSIS OF 3-ROLL METAL FORMING PROCESSES

by

J.J.M. Too

ABSTRACT

This report presents a mathematical formulation for the analysis of a 3-roll forming process. The derivation is based on the principle of energy balance; and on the assumptions of a rigid plastic material and a plane strain condition. The validity of the formulation is thus dependent on the closeness of the assumptions to the real situation.

INTRODUCTION

Considerable development in the analysis of 2-roll forming of metal sheets has been made by various investigators [e.g., 1-4], and a number of analytical studies on metal sheet bending have also been reported in the open literature [e.g., 1,5]. However, little has been done analytically on the 3-roll cylinder forming process, even though the process is common.

The objective of this report is to derive a set of formulae for the use of strain, power and efficiency calculations specifically for the pinch-type 3-roll forming process. The derivation is based on the principle of energy balance. Consideration is given to include the combined plastic bending and thinning effects in the formulation. Assumptions are introduced to reduce the complexity of the problem so that the resulting formulae can be applied with ease by design engineers.

Description of the problem

The mechanism of a pinch-type 3-roll forming process is depicted in Fig. 1. As shown in the figure, metal plates are fed into the system by the counter rotating action of the front rolls (rolls #1 and #2). These rolls transfer the energy to the plate via surface friction. A certain amount of pressure must be applied to both surfaces of the plate in order to provide sufficient frictional grip on the plate. The applied pressure can cause an elastic or plastic deformation of the plate and the rolls, depending upon the magnitude of the pressure and the material strength of both the plate and rolls. A permanent reduction of plate thickness can occur if the applied pressure is large. As the plate passes through the gap between the front rolls, it moves slower at the entrance A, and faster at the exit B, than the tangential speeds of the front rolls. A neutral point, N, can thus be found between points A and B. Because of the difference of flow speed between A and B, a tensile strain is introduced in the region. As the metal flows forward, the back roll (roll #3) imposes a bending action on the plate and produces a flexure strain. The plate is consequently formed into a cylinder.

It is possible, in thicker plate rolling, to apply a slightly faster rotating speed on roll #2 than roll #1. This introduces a larger flexure strain on the bottom surface than on the top surface of the plate, and produces a curvature before the plate is in contact with roll #3.

Because the process involves large deformation and straining, and the material properties can be temperature and stress dependent, the problem is thus highly non-linear and complex. To simplify the analysis, the following assumptions are made throughout the derivation:

- the material is rigid plastic, and elastic strains are negligible,
- the plane strain condition applies,
- variation of yield strength due to temperature change is negligible,
- shear stress across the thickness is ignored, and
- roll-flattening and bending are negligible.

The first two assumptions are justifiable because the elastic deformation usually is small in contrast with plastic deformation, and the strain in the axial direction is small. The third assumption is only valid approximately for a cold rolling, whereas the fourth assumption holds true only for a thin plate rolling. Roll flattening and bending normally have significant effects on the rolling process. They are ignored here purely for simplicity.

As mentioned, energy is transferred from the rolls to the plate by friction over the contacting surfaces. Part of this energy is dissipated through friction loss. Only a portion of the energy contributes to the plate deformation. This portion of the energy can be expressed as internal strain energy. The total work performed on the system is therefore the sum of the internal strain energy due to plate deformation and the friction loss over the contacting surfaces. This principle of energy and work balance forms the base of the derivation to follow.

Mathematical formulation

Using the Von Mises yield criteria and the assumption of a rigid plastic material, the internal strain energy can be written in a general form as

$$W_i = \frac{2}{\sqrt{3}} \sigma_y \int_V \sqrt{\frac{1}{2} \epsilon_{ij} \epsilon_{ij}}\, dV \; ; \quad \dot{W}_i = \frac{dW_i}{dt} \qquad \text{Eq 1}$$

where σ_y is the yield strength of the material, V is the volume of the plate, and ϵ_{ij} is the strain tensor. In 3-roll forming, W_i consists of two parts, namely, work done in plate thinning and bending. For the former, we further assume that the directions of principal strains are coincident with the X and

Y axes as shown in Fig. 1. It can be shown [6] that the strain components, in this case, are related to the plate thickness by the expressions:

$$\epsilon_{yy} = \ln \frac{h_b}{h_a}$$

$$\epsilon_{xx} = \ln \frac{h_a}{h_b} \qquad \text{Eq 2}$$

$$\epsilon_{zz} = 0$$

where h_a and h_b are the plate thickness at point A and point B respectively.

The internal strain energy per unit time from plate thinning can be obtained by substitution of Eq 2 into Eq 1 and integrating:

$$\dot{W}_i^T = \frac{2}{\sqrt{3}} \sigma_y h_b V^b \ln \frac{h_a}{h_b} \qquad \text{Eq 3}$$

where V^b is the speed at point B.

The flexure strain due to bending is simply

$$\epsilon_{xx} = \frac{y}{R_c} \qquad \text{Eq 4}$$

where y is the distance from the neutral surface of the plate, and R_c is the radius of curvature at the neutral surface, produced by the bending action. R_c is approximately equal to $1/2(R_o + R_i)$. R_o and R_i are the outer and inner radii of the cylinder being formed respectively.

Again, substituting Eq 4 into Eq 1, and integrating, one obtains the internal strain energy per unit time due to bending:

$$\dot{W}_i^B = \frac{2\sigma_y}{\sqrt{3}} V_3^r \int_0^{\frac{h_b}{2}} 2\frac{y}{R_c}\, dy$$

$$= \frac{\sigma_y}{\sqrt{3}} \frac{h_b^2}{R_o + R_i} V_3^r \qquad \text{Eq 5}$$

In Eq 5, V_3^r denotes the tangential speed of roll #3.

Note that not all the external work performed on the system is used in the plastic deformation. A significant part of the energy is dissipated by friction over the contact surfaces in regions II and III (see Fig. 1). This dissipated energy can be measured by the speed differences between the rolls and the plate surfaces (ΔV_1 and ΔV_2).

Let the speeds at the top and bottom surfaces of the plate be V_1 and V_2 respectively, and the tengential speeds of rolls 1, 2 and 3 by V_1^r, V_2^r, and V_3^r. Because the volume of material must remain constant for a plastic deformation, we can write, for region II:

$$\frac{V_1 + V_2}{2} h = \frac{V_1^n + V_2^n}{2} h_n = \frac{V_1^b + V_2^b}{2} h_b \qquad \text{Eq 6}$$

where the superscripts/subscripts b and n indicate the locations B and N; h denotes the thickness at a point in region II.

The surface moves at the same speed as the rolls at the neutral point N, i.e.,

$$V_1^n = V_1^r, \text{ and } V_2^n = V_2^r$$

We thus have

$$V_1 + V_2 = (V_1^r + V_2^r) \frac{h_n}{h} \qquad \text{Eq 7}$$

At point b, we have

$$V^b = \frac{V_1^b + V_2^b}{2} = \left(\frac{V_1^r + V_2^r}{2} \right) \frac{h_n}{h_b} \qquad \text{Eq 7a}$$

The thickness at a point in region II can be obtained directly from the geometry as shown in Fig. 1.

$$h = h_b + 2R_r (1 - \cos 2\alpha) \qquad \text{Eq 8}$$

where α is the angle of contact and R_r is the radius of rolls (assuming all rolls have the same size). For small α, $\sin \alpha \cong \alpha$ and $\cos 2\alpha \cong 1 - \alpha^2/2$. Equation 8 then becomes:

$$h \cong h_b + R_r \alpha^2 \qquad \text{Eq 9}$$

At the neutral point N,

$$h_n \cong h_b + R_r \alpha_n^2 \qquad \text{Eq 10}$$

Substituting Eq 9 and 10 into Eq 7, one obtains:

$$(V_1 + V_2) \cong (V_1^r + V_2^r) \left[\frac{h_b + R_r \alpha_n^2}{h_b + R_r \alpha^2} \right] \qquad \text{Eq 11}$$

subtracting both sides of Eq 11 from $(V_1^r + V_2^r)$ results in the sum of the speed differences on the top and bottom surfaces of the plate:

$$(\Delta V_1 + \Delta V_2) = (V_1^r + V_2^r) \left[\frac{h_b + R_r \alpha_n^2}{h_b + R_r \alpha^2} - 1 \right] \qquad \text{Eq 12}$$

The dissipated energy due to friction in region II can now be written as:

$$W_f \cong \int_s (\tau_1 + \tau_2)(\Delta V_1 + \Delta V_2) ds; \quad \dot{W}_f \cong \frac{dw_f}{dt} \qquad \text{Eq 13}$$

where τ_1 and τ_2 are the frictions on the top and bottom surfaces, and s is the surface area where plate and rolls are in contact.

Again, using von Mises yield criterion, the friction, τ can be expressed in terms of yield strength,

$$\tau_1 = \frac{m_1}{\sqrt{3}} \sigma_y$$

$$\tau_2 = \frac{m_2}{\sqrt{3}} \sigma_y \qquad \text{Eq 14}$$

m_1 and m_2 are the frictional factors representing the surface characteristics of the areas of contact of rolls 1 and 2 respectively.

Similar formulation can be written for the frictional loss at the contact area between roll 3 and the bottom of the plate. It is ignored in this case because the contact area is considered small.

Introducing Eq 12 and 14 into Eq 13, the dissipated energy per unit time becomes

$$\dot{W}_f = \frac{R_r \sigma_y}{\sqrt{3}} (m_1 + m_2) \int_0^{\alpha_n} (V_1^r + V_2^r) \left[\frac{h_b + R_r \alpha^2}{h_b + R_r \alpha_n^2} - 1 \right] d\alpha$$

$$- \int_{\alpha_n}^{\alpha_a} (V_1^r + V_2^r) \left[\frac{h_b + R_r \alpha_n^2}{h_b + R_r \alpha^2} - 1 \right] d\alpha \qquad \text{Eq 15}$$

where α_a is the contact angle covering region II. Integration of Eq 15 gives:

$$\dot{W}_f = \frac{R_r \sigma_y}{\sqrt{3}} (m_1 + m_2)(V_1^r + V_2^r) \left\{ \sqrt{\frac{R_r}{h_b}} \left(\frac{h_b}{R_r} + \alpha_n^2 \right) \left[2\tan^{-1} \sqrt{\frac{R_r}{h_b}}\, \alpha_n - \tan^{-1} \sqrt{\frac{R_r}{h_b}}\, \alpha_a \right] - (2\alpha_n - \alpha_a) \right\} \qquad \text{Eq 16}$$

It can be shown [7] that the contact angle α_a and α_n can be approximated by the expressions:

$$\alpha_a \cong \sqrt{\frac{h_a - h_b}{R_r}} \qquad \text{Eq 17}$$

where it is assumed that

$$\alpha_n \cong \frac{1}{2}\alpha_a$$

Introducing Eq 7a and 10 into Eq 3, and Eq 17 into Eq 16, we finally obtain:

$$\dot{W}_i = \dot{W}_i^T + \dot{W}_i^B$$

$$= \frac{1}{\sqrt{3}}\sigma_y h_b \left[2\left(1 + \frac{R_r \alpha_n^2}{h_b}\right)\left(\frac{V_1^r + V_2^r}{2}\right) \ln\frac{h_a}{h_b} + \frac{h_b V_3^r}{R_o + R_i} \right] \qquad \text{Eq 18}$$

and

$$\dot{W}_f = \frac{1}{\sqrt{3}}\sigma_y R_r (m_1 + m_2)(V_1^r + V_2^r)\, x$$

$$\left\{ \sqrt{\frac{h_b}{R_r}}\; 2\tan^{-1}\sqrt{\frac{h_a - h_b}{4h_b}} - \tan^{-1}\sqrt{\frac{h_a - h_b}{h_b}} \right\} \qquad \text{Eq 19}$$

Equation 18 represents the internal strain per unit time due to plastic deformation, and Eq 19 represents the energy per unit time dissipated over the surfaces in contact. Therefore, the total work performed per unit time on the system is the sum of Eq 18 and 19,

$$\dot{W} = \dot{W}_i + \dot{W}_f \qquad \text{Eq 20}$$

The efficiency of the 3-roll forming process is the percentage ratio of the work done on the plastic deformation to the total work performed on the system,

$$\eta = \frac{\dot{W}_i}{\dot{W}} \times 100\% \qquad \text{Eq 21}$$

Equations 2, 4, 18, 20 and 21 will permit the capability of the analysis of plastic strains, power, and efficiency for the pinch-type, 3-roll forming process with certain approximations as described in the text.

Sample problem

A sample problem is given here to show the application of the formulations presented above.

A plate has an original thickness of 2.2 in., with a yield strength of 36 Ksi. The plate is fed into a 3-roll forming machine for making a 20 in. radius cylinder. The gap between the 2 front rolls is 2 in. All 3 rolls are 17.8 in. in radius. The tangential speed of roll #1 is 6 in./min, and the speeds of rolls #2 and #3 are both 6.1 in./min. The frictional factors for all rolls are assumed to be 0.3. It is required to calculate the plastic strains from both plate thinning and bending, and the power and efficiency for the process of cylinder production.

The dimensions and parameters provided are listed below for the convenience of calculation:

$h_a = 2.2$ in. $\qquad \sigma_y = 36{,}000$ psi

$h_b = 2.0$ in. (gap) $\qquad V_1^r = 6$ in./min

$R_c = 20.0$ in. $\qquad V_2^r = V_3^r = 6.1$ in./min

$R_r = 17.8$ in. $\qquad m_1 = m_2 = m_3 = 0.3$

The outer and inner radii of the cylinder are respectively:

$$R_o = R_c + \frac{h_b}{2} = 21$$

$$R_i = R_c - \frac{h_b}{2} = 19$$

The strains due to plate thinning are calculated using Eq 2:

$$\epsilon_{xx} = -\epsilon_{yy} = \ln\frac{h_a}{h_b}$$

$$= 0.0953$$

$$\epsilon_{zz} = 0.0$$

The flexure strain at the outer surface of the cylinder is obtainable from Eq 4.

$$\epsilon_{xx} = \frac{h_b}{2R_c} = \frac{1}{20} = 0.05$$

The power required for plastic deformation can be obtained by Eq 18:

$$\dot{W}_i = \frac{\sigma_y h_b}{\sqrt{3}} \left[2 \left(1 + \frac{R_r \alpha^2}{h_b} \right) \left(\frac{V_1^r + V_2^r}{2} \right) \ln \frac{h_a}{h_b} + \frac{h_b V^r}{R_o + R_i} \right]$$

$$= \frac{36{,}000 \times 2}{\sqrt{3}} \left[2 \times 1.025 \times 6.05 \times \ln \frac{2.2}{2} + \frac{2 \times 6.1}{40} \right]$$

$$= 55{,}288.67 \text{ in.lb/min/in.}$$

The total work performed on the system can be calculated by Eq 20,

$$\dot{W} = \dot{W}_i + \frac{2}{\sqrt{3}} \sigma_y R_r (m_1 + m_2)(V_1^r + V_2^r) \times$$

$$\left\{ \sqrt{\frac{h_b}{R_r}} \; 2\tan^{-1} \sqrt{\frac{h_a - h_b}{4h_b}} - \tan^{-1} \sqrt{\frac{h_a - h_b}{h_b}} \right\}$$

$$= 55{,}288.67 + \frac{1}{\sqrt{3}} \times 36{,}000 \times 17.8 \times (0.6) \times 12.1 \times$$

$$\left\{ \sqrt{\frac{2}{17.8}} [2\tan^{-1} 0.158114 - \tan^{-1} 0.316228] \right\}$$

$$= 62{,}108.31 \text{ in.lb/min/in.}$$

The efficiency of the 3-roll forming process, for the conditions given, is

$$\eta = \frac{\dot{W}_i}{\dot{W}} \times 100\%$$

$$= \frac{55{,}288.67}{62{,}108.31} \times 100\%$$

$$= 89\%$$

Conclusion

A mathematical formulation for three-roll metal forming has been developed based on certain assumptions to make the problem more tractable. However, it can be used as a tool for evaluating power and machine efficiency with reasonable accuracy. A more complex approach must be used if greater precision is required.

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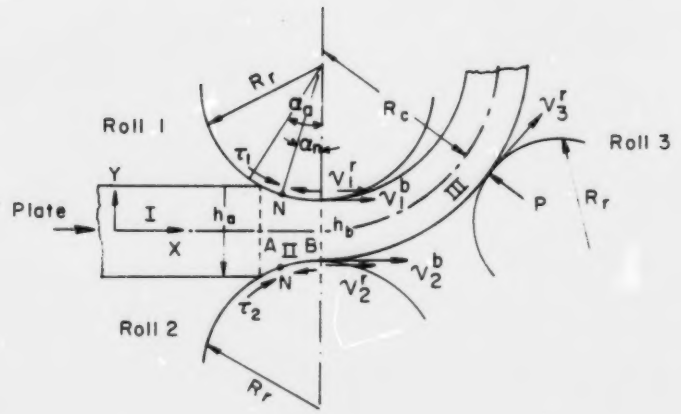

Fig. 1. A 3 roll forming process. (neg 7829)